Equations for H-Line Analysis in Mavis

Using displacements of sub-image elements to gather 3D and/or motion information is called optical flow analysis. When the camera's motion is known and is limited to straight-line motion, it's possible to use optical flow with line segments in a straightforward way for object avoidance in a mobile robot. The equations for doing that are derived below.

Camera model

A pinhole camera model with focal length *f* is assumed. This is shown in Figure 1. The image axis is the u,v axis, with its origin at the image center. Camera coordinates are X,Y,Z.

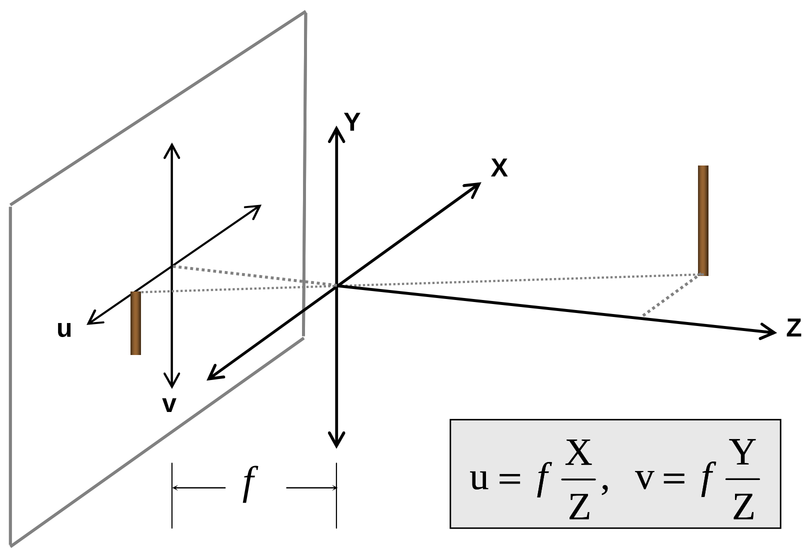

Figure 1. Pinhole camera model.

Image coordinates are related to the camera coordinates by similar triangles as u/*f* = X/Z, and v/*f* = Y/Z. This is usually written as

$$u = f\,\frac{X}{Z},$$

$$v = f\,\frac{Y}{Z}.$$

The camera is mounted on a robot and centered laterally. The robot's coordinate system is *x*,*y*,*z*. This coordinate system and the camera coordinate system share a common origin. Also, the robot's *x* axis is the same as the camera's X axis. The camera is rotated downward by a tilt angle θ. Thus, the camera's coordinates are related to the robot's coordinates by

$$X = x,$$
$$Y = y\cos(\theta) - z\sin(\theta),$$
$$Z = z\cos(\theta) + y\sin(\theta).$$

Height and distance

The first step is to compute the height of the 3D line segment and the distance from the robot's current location to where the line segment, or its continuation, intersects the robot's current trajectory. For this step, the line segment is assumed to be horizontal in 3D space. That assumption will be evaluated later. It's also assumed that the robot has traveled a known distance, *d*, in a straight line between image frames 1 and 2.

The setup for this step is shown in Figure 2. The point P in Figure 2 is the point of intersection between the robot's trajectory and the 3D line segment. Because the robot's motion is linear, this point on the line segment is located at u=0 in image frames 1 and 2. Since the robot and camera coordinate systems are attached to the camera, the travel motion changes the location of point P from z_1, its distance in frame 1, to z_2, its distance in frame 2. In the image coordinate system, the location of this point changes from $(0,v_1)$ to $(0,v_2)$, where v_1 and v_2 are the v coordinates of the line segment at u=0 in frames 1 and 2.

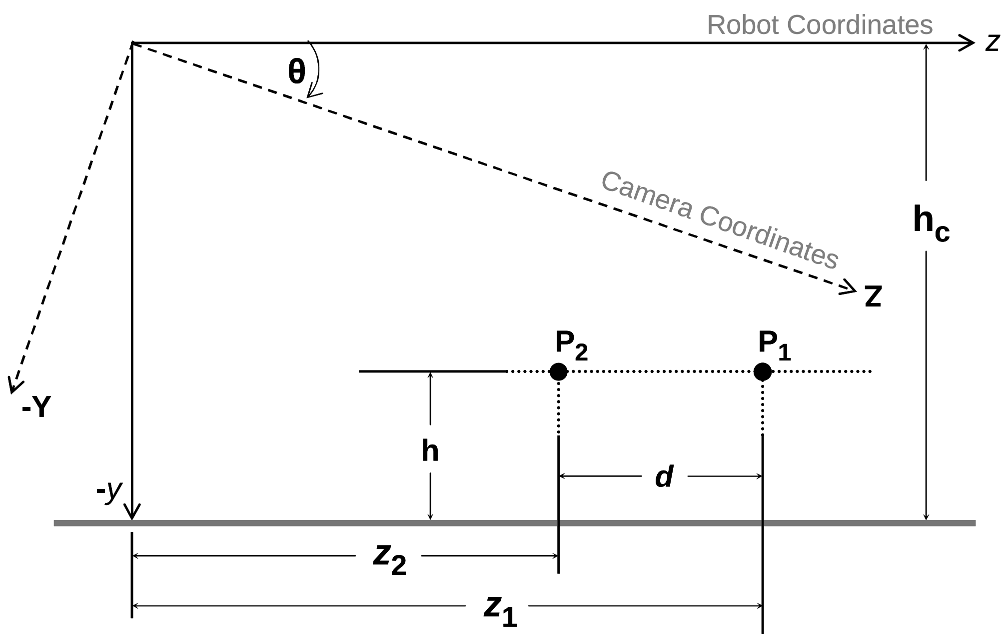

Figure 2. Location of the 3D line. P_1 is the intersection point in frame 1. P_2 is the same point in frame 2.

From the camera model,

$$v_1 = f\frac{Y_1}{Z_1},$$

$$v_2 = f\frac{Y_2}{Z_2}.$$

Substituting for Y and Z,

$$v_1 = f\,\frac{y\cos(\theta) - z_1\sin(\theta)}{z_1\cos(\theta) + y\sin(\theta)},$$

$$v_2 = f\,\frac{y\cos(\theta) - z_2\sin(\theta)}{z_2\cos(\theta) + y\sin(\theta)},$$

where
y = the y component of P in frames 1 and 2. These are equal for a line that's horizontal in 3D.

Replacing z_1 with $z_2 + d$ gives

$$v_1 = f\,\frac{y\cos(\theta) - (z_2 + d)\sin(\theta)}{(z_2 + d)\cos(\theta) + y\sin(\theta)},$$

$$v_2 = f\,\frac{y\cos(\theta) - z_2\sin(\theta)}{z_2\cos(\theta) + y\sin(\theta)}.$$

This yields two equations with two unknowns, y and z_2. Rearranging:

$(A_1 + D)\, z_2 + (B_1 - C)y = -d(A_1 + D),$
$(A_2 + D)\, z_2 + (B_2 - C)y = 0,$

where
$A_1 = v_1\cos(\theta),$
$B_1 = v_1\sin(\theta),$
$A_2 = v_2\cos(\theta),$
$B_2 = v_2\sin(\theta),$
$C = f\cos(\theta),$
$D = f\sin(\theta).$

Solving for z_2 in the second equation gives

$z_2 = (C - B_2)\, y\, /\, (A_2 + D).$

Substituting this back into the first equation and solving for y,

$y = d\,(A_1 + D)\, /\, K,$

where
$K = (C - B_1) + [(A_1 + D)(B_2 - C) / (A_2 + D)].$

The height of the 3D line is $h = h_c + y$.

Line orientation

The second step is to find the angle, φ, that the 3D line makes with the robot's *z* axis. As Figure 3 shows, this angle can be determined by finding the slope of the 3D line in the *x*, *z* plane of the robot's coordinate system. Then, $\varphi = \text{arccot}(-dz/dx)$. Various trig formulations could be used for this. The arccot function is stable when d*z* is at or near zero, which makes it a good choice numerically.

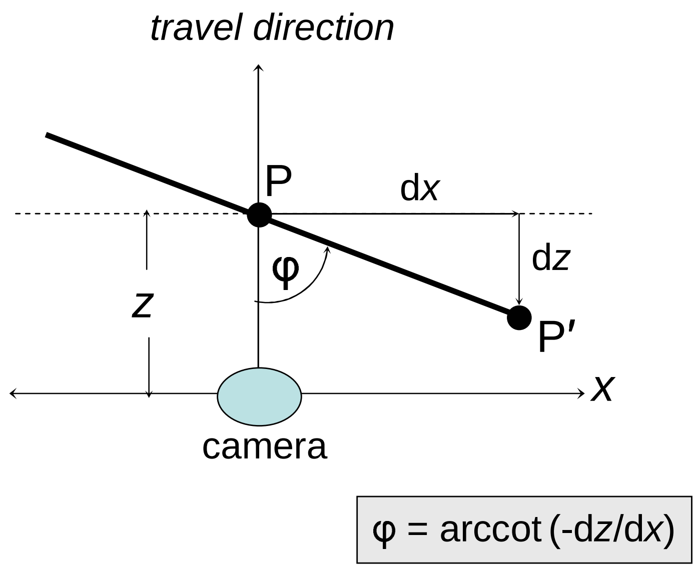

Figure 3. Angle, φ, that the 3D line makes with the robot's *z* axis.

The previous step established the location in space of a point, P, on the 3D line. As a convenience for computing d*z*/d*x*, take a hypothetical second point, P′, on the same line and locate it in 3D space as well. There's no special significance to P′. It's simply a computational device for finding d*z*/d*x*. It can be any point of the line, and it need not even be within the line segment's endpoints. A convenient point to use for this step is where the image line intersects the righthand edge of the image. The u,v coordinates of P′ are then (W/2, v′), where W is the image width.

To make this more concrete, let's use a particular image – frame 2. Then $v' = v'_2 = m_2(W/2) + v_2$ where m_2 is the slope of the line in frame 2 and v_2 is the value of v in frame 2 at u=0, as used in the previous step. From the camera model,

$$v'_2 = f\,\frac{Y'_2}{Z'_2}.$$

Substituting for Y'_2 and Z'_2,

$$v'_2 = f\,\frac{y\cos(\theta) - z'_2\sin(\theta)}{z'_2\cos(\theta) + y\sin(\theta)}.$$

There's only one unknown in this equation, z'_2. Solving for z'_2 gives

$$z'_2 = \frac{y(f\cos(\theta) - v'_2\sin(\theta))}{v'_2\cos(\theta) + f\sin(\theta)}.$$

Then dz = $z'_2 - z_2$. Solving for dx is similar. From the camera model,

$$u'_2 = \frac{W}{2} = f\,\frac{X'_2}{Z'_2}.$$

Using x = X and substituting for Z'_2,

$$\frac{W}{2} = \frac{f x'_2}{z'_2 \cos(\theta) + y \sin(\theta)}.$$

Solving for x'_2 gives

$$x'_2 = \frac{W}{2f}(z'_2 \cos(\theta) + y \sin(\theta)).$$

Then dx = $x'_2 - x_2 = x'_2$.

Checking the horizontal-line assumption

For the equations above, it's assumed that the 3D line is horizontal. To check that assumption, we compute φ in both frames 1 and 2. If the line is indeed horizontal, the two values for φ should be close. Intuitively, what this does is look at how the slope of the image line changes from frame 1 to frame 2 and validates that this change is consistent with a line that's horizontal in 3D.